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To Find The Comparison Of The Non Split Dominating, Induced Non Split Dominating Sets And The Distance, Radius, Eccentricity And Diameter Towards An Interval Graph G and $G^|$ from An Interval family

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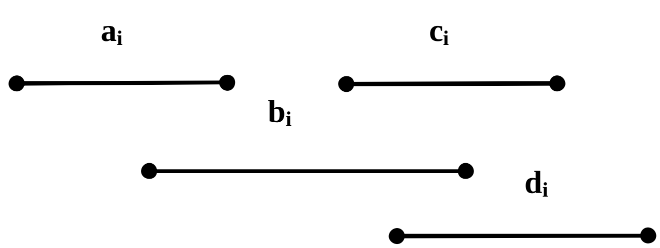
ABSTRACT: The diameter and the Radius of a graph are fundamental topological parameters that have many practical applications in real world networks. Interval graphs have a wide verity of applications to various branches of science and technology. Among the various applications of the theory of domination, the most often discussed is a communication network. In graph theory a connected component of un-directed graph is a sub graph in which any two vertices are connected to each other by paths. In this paper we discussed the comparison of the non split dominating, induced non split dominating sets and the distance, radius, eccentricity and diameter towards an interval graph G and $G^|$ from an interval family. Dia |V-DS| > Dia (G).

KEY WORDS: Non-Split dominating set, diameter, eccentricity, and radius, Distances.

I. INTRODUCTION

A graph G = (V,E) is an interval graph , if the vertex set V can be put into one to one correspondence with a set of intervals I. on the real line R, such that two vertices are adjacent in G if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I. The intervals $I = \{i_1, i_2, \ldots\ldots.. i_n\}$ be an interval family, where each i_i an interval on the real line and $i_i = [a_i, b_i]$ for i=1,2,…….n ,here a_i is called left end point labeling and b_i is the right end point labeling of i_i. Without loss of generality we assume that all end points of the interval in I are distinct numbers between 1 and 2n.

A simple graph G is an interval graph if it is an intersection graph corresponding to a family of intervals on the real line.



Interval family I

The research of the domination in graphs has been ever green of the graph theory. Its basic concepts are the dominating set and the domination number. The theory of domination in graphs was introduced by Ore (1) and Berge (2). A survey on results and applications of dominating set was presented by E.J.Cockayane and S.T.Hedetniemi (3). In 1997 V.R.Kulli et. all introduced the concept of non-split domination (4). Let G be a graph and V_1 be a subset of V. Then the subgraph of G whose vertex set is V_1 and edge set is the set of whose edges in E whose both ends are in V_1 is called the vertex induced subgraph and is denoted by $< V_1 >$. A subset DS of V is said to be a dominating set of G if every vertex in V\DS is adjacent to a vertex in DS. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G.
A dominating set DS of a graph G is a non - split dominating set if the vertex induced subgraph $< V - DS >$ is connected. The non - split domination number $\gamma_{ns}(G)$ of the graph G is the minimum cardinality of the non - split dominating set.

The diameter and radius are two of the most basic graph parameters. The diameter of a graph is the largest distance between its vertices. The distance orientations of graphs by V.Chvatal (5). The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by Diam (G) that is Diam (G) = max {e (v): v ЄV (G)}, where the maximum distance from vertex u to any vertex of G is called eccentricity of the vertex v and is denoted by ecc (V) that is ecc (v) = max {d (u,v): u ЄV (G)},where as the distance between two vertices u and v of a graph is the length of the shortest path between them and is denoted by d_G (u,v) or d (u,v).

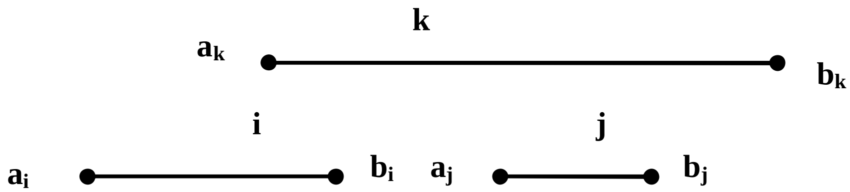
2.2 MAIN THEOREM

Theorem :

Let DS be a dominating set and D_i is a diameter of the given interval graph G. If a_i and a_j are any two intervals in I such that a_i Є DS, $a_j \neq 1$ and a_j is contained in a_i and if there is at least one interval to the left of a_j that intersects a_j and at least one interval $a_k \neq a_i$ to the right of a_j that intersects a_j then the non-split domination occurs in G and also Diam |<V-DS>| > Diam (G).

Proof:

Suppose there is at least one interval $a_k \neq a_i$ to the right of a_j and that intersects a_j. Then it is obvious that a_j is adjacent to a_k in <V-DS>, so that there will not be any disconnection in <V-DS>. Since there is at least one interval to the left of a_j that intersects a_j, there will not be any disconnection in <V-DS> to its left. Thus we get non-split domination in G. Next we will find the diameter of the given interval graph. Let us find the distance of the given interval family that is for two vertices a_i and a_j in a graph G, the distance from a_i to a_j is denoted by d (a_i,a_j) and defined as the length of a shortest (a_i-a_j)- path in graph G.



For a connected graph G the term distance we just define satisfies all four of the following properties.

1. $d\,(a_i,a_j) \geq 0$, for all $a_i,a_j \in V(G)$.
2. $d\,(a_i,a_j) = 0$, if and only if $a_i = a_j$.
3. $d\,(a_i,a_j) = d\,(a_j,a_i)$, for all $a_i,a_j \in V(G)$.
4. $d\,(a_i,a_k) \leq d\,(a_i,a_j) + d\,(a_j,a_k)$, for all $a_i,a_j,a_k \in V(G)$.

Suppose an interval graph V(G) is not a connected and then G_1 and G_2 are two graphs of G such that $G= G_1 U G_2$ and $E(G_1)UE(G_2)$. And $G_1 \cap G_2 = \Phi$ that is $V\,(G_1) \cap V\,(G_2) = \Phi$ and $E\,(G_1) \cap E\,(G_2) = \Phi$. Then $d\,(a_i,a_j) = \infty$ for $a_i \in V(G_1).a_j \in V(G_2)$. Then G must be connected.

And also we prove that the diameter of the graph towards the eccentricity of a vertex a_j denoted by ecc (a_j) is the distance from a_j to a vertex farthest from a_j. i.e;

$$ecc\,(a_j) = \max_{l \in VG}(a, l)$$

And the diameter of a graph is the maximum of its eccentricities, or equality the maximum distance between two vertices i.e;

$$Diam\,(G) = \max_{l \in VG}(ecc(l)) = \max_{l \in VG}\{d(l, m)\}$$

Also the Radius of a graph G, denoted Rad (G) is the minimum of the vertex eccentricities. i.e;

$$Rad\,(G) = \min_{l \in VG}\{ecc(l)\}$$

We know that any connected graph G, Rad (G) ≤ Dia (G) ≤ 2(Rad (G)) and finally we get induced sub graph Diam |V-DS| > Diam (G).

There fore the theorem is proved from the interval graph G.

2.3 ILLUSTRATIONS

II. LUSTRATION

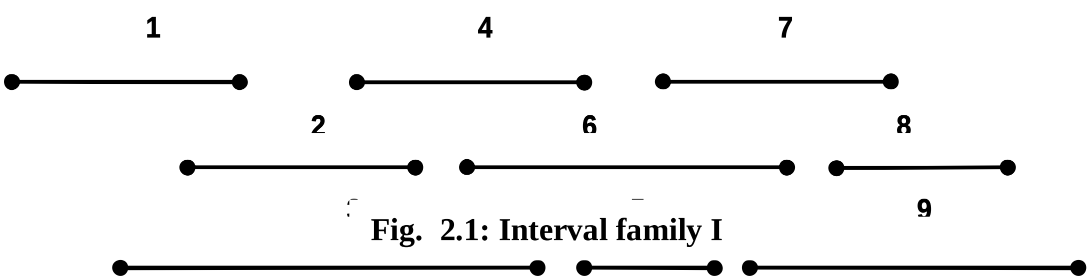


Fig. 2.1: Interval family I

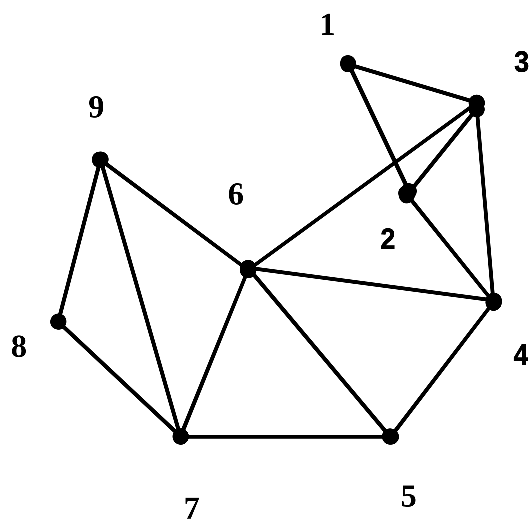


Fig. 2.2: Interval graph G

Dominating set of the interval graph is D = {3, 7} and the cardinality **|DS|=2**

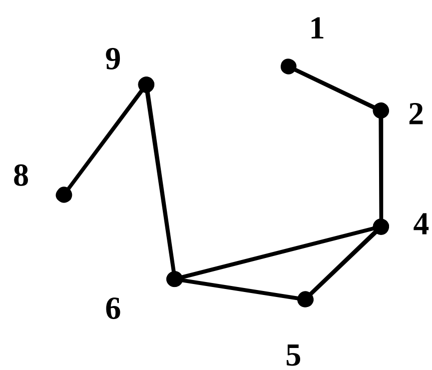


Fig. 2.3: vertex induced graph <V-DS>-connected graph G|

From an induced sub-graph DS = {2, 6, 8} in this the cardinality **| <V-DS> | = 3**

2.4 TO FIND DISTANCE, ECCENTRACITY, RADIUS AND DIAMETER FROM G

2.4.1 TO FIND THE DISTANCES FROM G

d (1,1)=0	d (2,1)=1	d (3,1)=1	d (4,1)=2	d (5,1)=3
d (1,2)=1	d (2,2)=0	d (3,2)=1	d (4,2)=1	d (5,2)=2
d (1,3)=1	d (2,3)=1	d (3,3)=0	d (4,3)=1	d (5,3)=2
d (1,4)=2	d (2,4)=1	d (3,4)=1	d (4,4)=0	d (5,4)=1
d (1,5)=3	d (2,5)=2	d (3,5)=2	d (4,5)=1	d (5,5)=0
d (1,6)=2	d (2,6)=2	d (3,6)=1	d (4,6)=1	d (5,6)=1
d (1,7)=3	d (2,7)=3	d (3,7)=2	d (4,7)=2	d (5,7)=1
d (1,8)=4	d (2,8)=4	d (3,8)=3	d (4,8)=3	d (5,8)=2

d (6,1)=2	d (7,1)=3	d (8,1)=4	d (9,1)=3
d (6,2)=2	d (7,2)=3	d (8,2)=4	d (9,2)=3
d (6,3)=1	d (7,3)=2	d (8,3)=3	d (9,3)=2
d (6,4)=1	d (7,4)=2	d (8,4)=3	d (9,4)=2
d (6,5)=1	d (7,5)=1	d (8,5)=2	d (9,5)=2
d (6,6)=0	d (7,6)=1	d (8,6)=2	d (9,6)=1
d (6,7)=1	d (7,7)=0	d (8,7)=1	d (9,7)=1
d (6,8)=2	d (7,8)=1	d (8,8)=0	d (9,8)=1

2.4.2 T

ecc [v (G)] = max {d (u,v): u ЄV(G)}

e(1)= max{d (1,1),d (1,2),d (1,3),d (1,4),d (1,5),d (1,6),d (1,7),d (1,8),d (1,9)}
= max {0, 1, 1, 2, 3, 2, 3, 4, 3} = 4

e(2)= max{d (2,1),d (2,2),d (2,3),d (2,4),d (2,5),d (2,6),d (2,7),d (2,8),d (2,9)}
= max {1, 0, 1, 1, 2, 2, 3, 4, 3} = 4

e(3)= max{d (3,1),d (3,2),d (3,3),d (3,4),d (3,5),d (3,6),d (3,7),d (3,8),d (3,9)}
= max {1, 1, 0, 1, 2, 1, 2, 3, 2} = 3

e(4)= max{d (4,1),d (4,2),d (4,3),d (4,4),d (4,5),d (4,6),d (4,7),d (4,8),d (4,9)}
= max {2, 1, 1, 0, 1, 1, 2, 3, 2} = 3

e(5)= max{d (5,1),d (5,2),d (5,3),d (5,4),d (5,5),d (5,6),d (5,7),d (5,8),d (5,9)}
= max {3, 2, 2, 1, 0, 1, 1, 2, 2} = 3

e(6)= max{d (6,1),d (6,2),d (6,3),d (6,4),d (6,5),d (6,6),d (6,7),d (6,8),d (6,9)}
= max {2, 2, 1, 1, 1, 0, 1, 2, 1} = 2

e(7)= max{d (7,1),d (7,2),d (7,3),d (7,4),d (7,5),d (7,6),d (7,7),d (7,8),d (7,9)}
= max {3, 3, 1, 2, 1, 1, 0, 1, 1} = 3

e(8)= max{d (8,1),d (8,2),d (8,3),d (8,4),d (8,5),d (8,6),d (8,7),d (8,8),d (8,9)}
= max {4, 4, 3, 3, 2, 2, 1, 0, 1} = 4

e(9)= max{d (9,1),d (9,2),d (9,3),d (9,4),d (9,5),d (9,6),d (9,7),d (9,8),d (9,9)}
= max {3, 3, 2, 2, 2, 1, 1, 1, 0} = 3

ecc (V)= max {e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9)}
= max {4, 4, 3, 3, 3, 2, 3, 4, 3} = 4

The eccentricity of vertices is ecc [v(G)] =4

2.4.3 TO FIND THE RADIUS FROM G

Rad (G) = min {ecc (v): v ЄV (G)}

Rad (G) = min {e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9)}
= min {4, 4, 3, 3, 3, 2, 3, 4, 3} = 2

Therefore Radius = 2

2.4.4 TO FIND THE DIAMETER FROM G:

Diam (G) = max {e (v): v ЄV (G)}

Diam(G) = max {e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9)}
= max {4, 4, 3, 3, 3, 2, 3, 4, 3} = 4

Therefore diameter = 4

In this connection we get an eccentricity equal to the Diameter ecc [v (G)] = Diam (G)

And also we can easily to prove that Rad (G) ≤ Diam (G) ≤ 2Rad (G)

2.5 TO FIND DISTANCE, ECCENTRACITY, RADIUS AND DIAMETER FROM AN IDUCED CONNECTED SUBGRAPH $G^{|}$

2.5.1 TO FIND THE DISTANCES FROM AN INDUCED CONNECTED SUBGRAPH $G^{|}$

d (1,1)=0	d (2,1)=1	d (4,1)=2	d (5,1)=3
d (1,2)=1	d (2,2)=0	d (4,2)=1	d (5,2)=2
d (1,4)=2	d (2,4)=1	d (4,4)=0	d (5,4)=1
d (1,5)=3	d (2,5)=2	d (4,5)=1	d (5,5)=0
d (1,6)=3	d (2,6)=2	d (4,6)=1	d (5,6)=1
d (1,8)=5	d (2,8)=4	d (4,8)=3	d (5,8)=3
d (1,9)=4	d (2,9)=3	d (4,9)=2	d (5,9)=2

d (6,1)=3	d (8,1)=5	d (9,1)=4
d (6,2)=2	d (8,2)=4	d (9,2)=3
d (6,4)=1	d (8,4)=3	d (9,4)=2
d (6,5)=1	d (8,5)=3	d (9,5)=2
d (6,6)=0	d (8,6)=2	d (9,6)=1
d (6,8)=2	d (8,8)=0	d (9,8)=1
d (6,9)=1	d (8,9)=1	d (9,9)=0

2.5.2 TO FIND AN ECCENTRICITY FROM INDUCED CONNECTED SUBGRAPH $G^{|}$

$$ecc\ [v(G^{|})] = \max\ \{d\ (u,v): u \in V(\mathbf{G}^{|})\}$$

e(1)= max{d (1,1),d (1,2),d (1,4),d (1,5),d (1,6), d (1,8),d (1,9)}
= max {0, 1, 2, 3, 3, 5, 4} = 5
e(2)= max{d (2,1),d (2,2),d (2,4),d (2,5),d (2,6),d (2,8),d (2,9)}
= max {1, 0, 1, 2, 2, 4, 3} = 4

e(4)= max{d (4,1),d (4,2), d (4,4),d (4,5),d (4,6),d (4,8),d (4,9)}
= max {2, 1, 0, 1, 1, 3, 2} = 3

e(5)= max{d (5,1),d (5,2),d (5,4),d (5,5),d (5,6),d (5,8),d (5,9)}
= max {3, 2, 1, 0, 1, 3, 2} = 3

e(6)= max{d (6,1),d (6,2),d (6,4),d (6,5),d (6,6),d (6,8),d (6,9)}
= max {3, 2, 1, 1, 0, 2, 1} = 3

e(8)= max{d (8,1),d (8,2),d (8,4),d (8,5),d (8,6),d (8,8),d (8,9)}
= max {5, 4, 3, 3, 0, 2, 1} = 5

e(9)= max{d (9,1),d (9,2), d (9,4),d (9,5),d (9,6),d (9,8),d (9,9)}
= max {4, 3, 2, 2, 1, 1, 0} = 4

ecc [v(G|)] = max {e(1), e(2), e(4), e(5), e(6), e(8), e(9)}
= max {5, 4, 3, 3, 3, 5, 4} = 5

The eccentricity of vertices is ecc [v(G|)] = 5

2.5.3 TO FIND THE RADIUS FROM AN INDUCED CONNECTED SUBGRAPH G|

Rad (G) = min {ecc (v): v ЄV (G|)}

Rad (G|) = min {e(1), e(2), e(4), e(5), e(6), e(8), e(9)}
= min {5, 4, 3, 3, 3, 5, 4} = 3

Therefore Radius = 3

2.5.4 TO FIND THE DIAMETER FROM AN INDUCED CONNECTED SUBGRAPH G|

Diam (G|) = max {ecc (v): v ЄV (G|)}

Diam (G|) = max {e(1), e(2), e(4), e(5), e(6), e(8), e(9)}
= max {5, 4, 3, 3, 3, 5, 4} = 5
Therefore diameter = 5

In this we have an eccentricity is equal to the Diameter ecc [v(G|)] = Diam (G|)

And also we prove that easily Rad (G|) ≤ Diam (G|) ≤ 2Rad (G|)

From the theorem any connected graph G, Rad (G) ≤ Diam (G) ≤ 2(Rad (G)) and an induced connected subgraph G|, the Diam |V-DS| (G|) > Diam (G|).

There fore the theorem is proved from G and G|.

III. CONCLUSION

The interval graphs are rich in combinatorial structure and have found applications in several disciplines such a Traffic control and computer science and particularly useful in real line scheduling and computer storage allocation problems in this paper we individualized an interval graphs as various graphs. We then extended the results to trace out a specific type of an interval graphs having every pair of vertices as the comparison of the non split dominating, induced non split dominating sets and the distance, radius, eccentricity and diameter towards an interval graph g and $g^{|}$ from an interval family

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