

# Delay Analysis of Wireless Sensor Networks using Queuing Theory

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**Abstract-** Ubiquitous wireless sensor network are expected to play an important role in the future society for various applications. As a result, carefully managing the network resources to improve the network performance becomes a hot research topic. The common challenges of a WSN are network connectivity, node mobility, energy consumption, data computation and aggregation at sensor nodes. In this paper we focus on intermittency in network connectivity due to mobility of sensor nodes. We propose a new mathematical model to capture a given entire WSN as is with intermittency introduced between the communication links due to mobility. The model involves open  $GI/G/N$  queuing networks whereby intermittency durations in communication links are captured in terms of mobility models. The analytical formulas for the performance measures such as average end-to-end delay, packet loss probability, throughput, and average number of hops are derived using the queuing network analyzer and expansion method for models with infinite and finite-buffer nodes, respectively.

**Keywords:** Intermittent Models Markov process, Mathematical Optimization, Mobility models, Queuing networks, Wireless sensor networks.

## I. INTRODUCTION

In this paper we propose a new model to capture a given entire WSN as is with intermittency introduced between the communication links due to mobility. The model facilitates the use of intermittency distributions of different mobility models [1, 2,10]. Our model is derived from open queuing networks-a special field of stochastic processes [3,4,9]. In this model, each sensor node of the WSN is viewed as a queuing node the wireless communication link between sensors nodes are viewed as intermittent links between queuing nodes. The derivation of the model involves the modification of existing open queuing networks by introducing intermittency among the links in order to capture the intermittency in network connectivity in WSN due to mobile nodes [5]. We present different variations of our model to capture different intermittency scenarios and nodes' buffer sizes of WSNs. We provide a comprehensive set of parameters to analyze WSNs with sparse mobile nodes, medium mobile nodes and high mobile nodes. To the best of our knowledge, there is no model to analytically study performance of mobile WSNs. The paper is structured as follows. We derive analytical formulas for performance measure of 2-state and multi-state intermittent models in section 2. In section 3, we provide numerical results for the performance measures of different networks. We conclude the paper in section 4.

## II. ANALYTICAL ANALYSIS OF QUEUING DELAY

### A. WSN Model with infinite buffer stage

Model with intermittent reception, each node's reception goes on and off with average rates  $\alpha$  and  $\beta$  and variances  $v_{on}$  and  $v_{off}$ . When the reception is on, the nodes receives data from other nodes, otherwise, the data are lost. Let  $\lambda'_j$  denotes the average arrival rate of data at node  $j$  due to its intermittent reception. We note that if there is no intermittency in the reception, the average arrival of data at node  $j$  is given by  $\lambda_j$ . (as in (1) with  $p_j^{(N)}=0$ ). Let  $C_{aj}^{I2}$  be the SCV of the intermittent arrival process of data at node  $j$ .

$$\lambda'_j = p_{on} \lambda_j \quad (6)$$

$$C_{aj}^{I2} = C_{aj}^2 + k \lambda_j \quad (7)$$

Where  $P_{on}$  is the probability that node  $j$  is receiving packets from other nodes and is given by

$$p_{on} = \frac{\beta}{\alpha + \beta} \quad (8)$$

$$k = \frac{\alpha(v_{on}\alpha^2 + v_{off}\beta^2)}{(\alpha + \beta)^2} \quad (9)$$

Model with intermittent transmission and / or reception each node's transmission and / or reception go on and off with average rates  $\alpha$  and  $\beta$  and variance  $v_{on}$  and  $v_{off}$ . When a node, say  $j$  is in the receiving distance of a node, say  $i$  we say that connectivity link is from  $i$  to  $j$  is on so that  $i$  can route packets to  $j$  with routing probability  $r_{ij}$ . Otherwise, we say that the link from  $i$  to  $j$  is off. When the link is off, all the routed packets are lost, let  $\lambda'_{ij}$ . Denote the average arrival rate of data at node  $j$  due to its intermittent link. We note that if the link are non-intermittent, the average arrival of data at node  $j$  is given by  $\lambda_j$  (as in (1) with  $P_j^{(N)}=1$ ) let  $C_{aj}^{I2}$  be the SCV of the intermittent arrival process of data at node.

$$\lambda_j = \lambda_{oj} + p_{on} \sum_{i=1}^M \lambda_{ij} \quad (10)$$

$$C_{aj}^{I2} = 1 - \omega_j 1 - \left( \sum_{i=1}^M p_{ij} c_{ij}^2 - k \sum_{i=1}^M p_{ij} \lambda_{ij} \right) \quad (11)$$

Where  $p_{on}$  is the probability that the link between any two nodes is on

Model with intermittent transmission the processing unit in each node goes on and off with average  $\alpha$  and  $\beta$  respectively, according to a general renewal processes. Independent of the intermittent state of the processing unit, the node continues to receive data from other reachable nodes. We recall that in this model there is no loss of packets as the nodes stop transmitting packets whenever they go out of transmission range. Let  $\mu_j$ . denote the average processing rate of data at node  $j$  due to its intermittency. We note that if the links are non-intermittent, the average processing rate of data at node  $j$  is given by  $\mu_j$ . Let  $C_{sj}^{I2}$  be the SCV of the intermittent processing time of data at node  $j$ .

### B. WSN Model with finite buffer stage

In finite buffer models, an artificial node is added to each sensor node model. Whenever the node's buffer is full, the arriving packet at a sensor node is route to the associated artificial node. We note that the algorithm for the expansion method is derived on the assumption that after incurring a random /deterministic delay, the packet in the artificial node attempts to rejoin the buffer and if the buffer is full, it is routed back to the artificial node. This process continues for the packets in the artificial node until the artificial node is empty. We modify this assumption so that packets that join artificial node do not try to join the node's buffer and hence they are considered to be lost due to buffer overflow. Model with intermittent reception  $I$  is the same as the one analyzed with infinite buffer nodes except that each node has a finite buffer of size  $N$ . Let  $\lambda'_j$  denote the average arrival rate of data at node  $j$  due to its intermittent reception. Out of these successfully arrived packets, some of the packets are lost due to buffer overflow at node  $j$ . Hence let  $\lambda''_j$  be the effective arrival rate of data which successfully joined the buffer of node  $j$ . Let  $C_{aj}''^2$  be the SCV of the arrival process of packets who joined the buffer of node  $j$ .

$$\lambda''_j = (1 - p_N^{(j)}) \lambda'_j \quad (14)$$

$$C_{aj}''^2 = y_j \left[ (1 - p_N^{(j)}) c_{aj}'^2 + p_N^{(j)} \right] + 1 - y_j \quad (15)$$

Model with intermittent transmission and / or reception is the same as the one analyzed with infinite buffer node model except that each node has a finite buffer of size  $N$ . Let  $\lambda'_j$  denote the average arrival rate of data at node  $j$  due to its intermittent transmission and / or reception. Out of these successfully arrived packets, some of the packets are lost due to buffer overflow at node  $j$ . Hence  $\lambda''_j$  let be the effective arrival rate of data which successfully joined the buffer of node  $j$ . Let  $C_{aj}''^2$  be the SCV of the arrival process of packets who joined the buffer of node  $j$ .

$$\lambda''_j = (1 - p_N^{(j)}) \left( \lambda_{oj} + \sum_{i=1}^M \lambda'_{ij} \right) \quad (16)$$

$$C_{aj}''^2 = y_j'' (1 - p_N^{(j)}) \left[ y_j' \sum_{i=1}^M \left\{ \frac{\lambda'_{ij}}{\sum_{k=1}^M \lambda'_{kj}} \right\} c_{ij}'^2 + 1 - y_j' \right] + \lambda_j'' p_N^{(j)} + 1 - \lambda_j'' \quad (17)$$

Now we consider the average number of hops in this section, we consider a WSN model with 2-state or multi-state intermittent communication channels and with equal routing probabilities, communication channels and with equal routing probabilities, say  $r_j=r, \forall_i, j=1,2, \dots, M$ . This assumption is valid for scenarios whereby all sensor nodes are in the communication ranges among themselves and hence any sensor node has a equal routing probability of routing the packet to any other node. For this model, we provide the average number of hops in the following theorem.

**Theorem:** For the WSN model described above, the average number of hops  $H_A$  a packet goes through the WSN before it reaches the sink is given by

$$H_A = \frac{1}{1 - Mr} \quad (18)$$

Where  $M$  is the number of sensor nodes. Let  $H$  denote the number of hops a packet, generated by sensor nodes, goes through the WSN before it reaches the sink. Then

$$pr\{H = h\} = \left( \sum_{j=1}^M r_{ij} \right)^{h-1} \left( 1 - \sum_{j=1}^M r_{ij} \right) \quad (19)$$

$$= (Mr)^{h-1} (1 - Mr) \quad (20)$$

Note that the above formula is valid only for the case where  $r_{ij}$  are same for all  $i, j=1, 2, \dots, M$ . The average number of hops  $H_A$  is then given by

$$H_A = \sum_{h=1}^{\infty} h \Pr\{H = h\} = \frac{1}{1 - Mr} \quad (21)$$

### III. NUMERICAL RESULTS

In this section we produce some numerical results for the performance measures of the intermittent network models for different values of  $N$ ,  $M$ ,  $K$ , and different distribution for intermittent, arrival and processing time. Since infinite buffer models are special cases of the finite buffer model with multi-state intermittent links, we provide numerical results of this generic finite buffer model with multi-state intermittent links. our main focus of our paper is to analyze the performance measures such as packet loss probability and average end-to-end delay in the network due to intermittency in the communication channel, we vary the following four major parameters which play a crucial role in terms of network connectivity: (i) number of intermittent states  $K$  of a communication channel, (ii) average number of hops  $H_A$ , (iii) buffer size  $N$ , and (iv) data sample time of a node. We note that the density of a given WSN, for a fixed topology and radio range of all nodes, can be captured in terms of  $H_A$  the smaller  $H_A$  values correspond to dense networks and its larger values correspond to sparse network and its larger values correspond to sparse, the packets would reach the sink in less number of hops whereas for a sparse network, the packets are routed through other nodes before they reach the sink.

In fig (3,4) we plot the performance measures total packet loss probability and average end-to-end delay, for different values of links intermittent state  $K$  and average number of hops  $H_A$ , a generated packet goes through before reaches the sink. It is clear from fig. 3, the number of state  $K$  of the intermittent state increases, the packet loss probability  $P_L$  decreases and reaches the minimum when  $K = 14$ . This is the gain we wanted to achieve by introducing multi-state link due to its practicality when compared to 2-state intermittency. As the average number of hops  $H_A$  increases,  $P_L$  increases due to intermittency in the link. When the number of hops increases, packets are being forwarded to intermediate nodes before they reach the sink. During this forwarding process, some packets are lost due to intermittent links and hence loss due to intermittency  $P_L$  increases as  $H_A$  increases. It is interesting to get insight of these WSNs by able to answer questions like what should be the number of intermittent states and averages number of hops in order to achieve not more than 50% packet loss by referring fig 4. When the loss of packets decreases, the number of packets in the network increases and as a result the end-to-end delay increases.

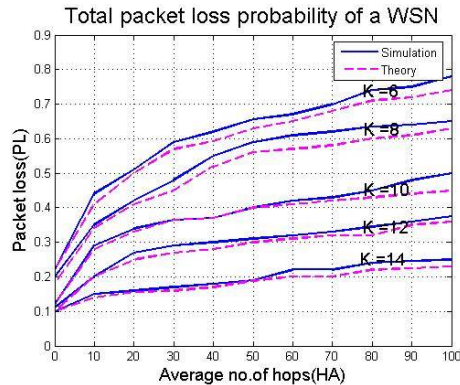


Fig. 3 Total packet loss probability of a WSN

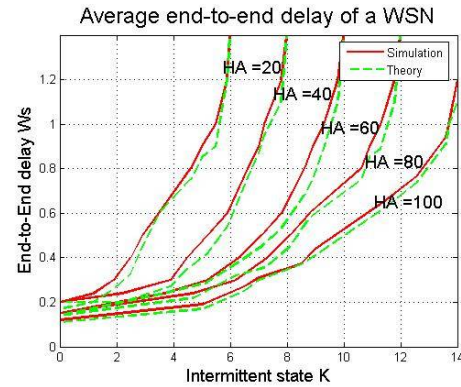


Fig. 4 Average end-to-end delay of a WSN

We consider a WSN model with 20 finite buffer nodes and 14-state communication links with Rayleigh distributed intermittent transmission and / or reception. We assume that at each node, packets are generated according a Poisson process with average generation rate of 6 packets per second and deterministic processing time of 0.035 sec/packet. In fig (5,6) we plot the performance measures total packet loss probability, packet loss due to intermittency and finite buffer, and average end-to-end delay, respectively, for different buffer size  $N$  of nodes and average number of hops  $H_A$  a generated packet goes through before the sink. See the effect of intermittency in packet loss, probability is plotted for different averages of number of hops inn Fig 5. The  $P_L$  increases from 0 to 0.9 as the average number of hops increases from 1 to 140 when the buffer size increases from 1 to 20. This is due to 100% load in the network which is nothing but the average of the ratios  $\frac{\lambda'_j}{\mu_j}$  over all  $j$ . Due to this heavy traffic, the  $P_L$  gradually reduces as  $N$  due to intermittency whereas the loss due to finite buffer  $P_{Lbuf}$  keeps increasing linearly due to more and more packets are being routed.  $H_A$  increases from 1 to 30 and stabilizes where as  $P_{Lbuf}$  remains zero initially. From fig. 6 we understand that though we lose more packets due to intermittency as  $H_A$  increases, the average end-to-end delay  $Ws$  increases exponentially for large fixed  $N$ . This is because as the buffer size increases, packets are being buffered due to 100% traffic load in the network and hence causes more delay. Whereas when  $N=1$ ,  $Ws$  reaches steady-state as  $H_A$  increases.

This is done see the effect of buffer size and sampling times on the performance measures. In the sampling time region  $[0.1, 0.4]$  the  $P_L$  is more when compared to its value in the sampling time region  $[0.4, 0.8]$  where it is almost zero. This is because in the sampling time region  $[0.1, 0.4]$ , the load of the network is more than 100%. This effect can be seen in the average end-to-end delay  $Ws$ 's graph in fig 8. During the heavy traffic sampling time region  $[0.1, 0.4]$ , the  $Ws$  increases linearly as the buffer size increases as more and more packets join the buffer. In this region, the  $Ws$  increases as the sampling time increases and reaches respective local maximums when the sampling time is 0.32 and starts decreasing to 1 around 0.5. It reaches the maximum value of 22 seconds when  $N = 100$ .



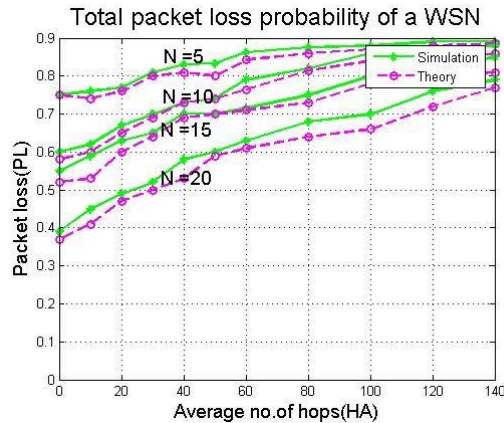


Fig. 5 Total packet loss probability of a WSN

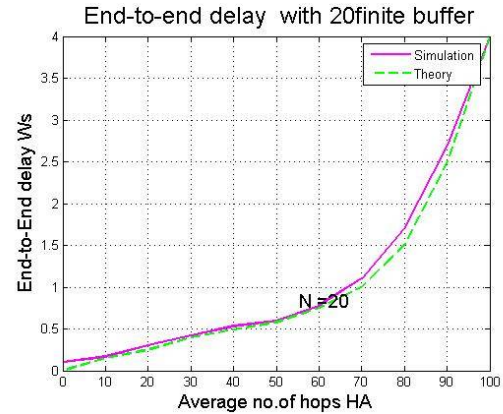


Fig. 6 Average end-to-end delay of a WSN

#### IV. CONCLUSION

We modeled WSN using suitable stochastic networks known as open queuing networks and derived analytical formulas for the performance measures such as throughput, average end-to-end delay, and packet loss probability. These mathematical models are amenable to use intermittent distributions of communication links based on mobility models. Extensive numerical results are carried out to illustrate the efficiency of the analytical results. Mathematical models are available in the literature which can possibly be used to model an individual wireless sensor node but as far as we are aware, this is the first attempt to model entire mobile WSNs by using existing mobility models. The model does not put any limitations on the parameter values or the distributions to capture packet generation, processing time and intermittency. Our focus was mainly on the intermittency in network communication among sensor nodes due to mobility. We assumed the rest are assumed to function well. Comparison of the analytical and real time with the simulation results of WSN simulation as shown in the fig. (9 & 10). The subject of further interest includes modeling the retransmission of packets and medium access control algorithms, protocols and compare the analytical results with the simulation results of WSNs packages.

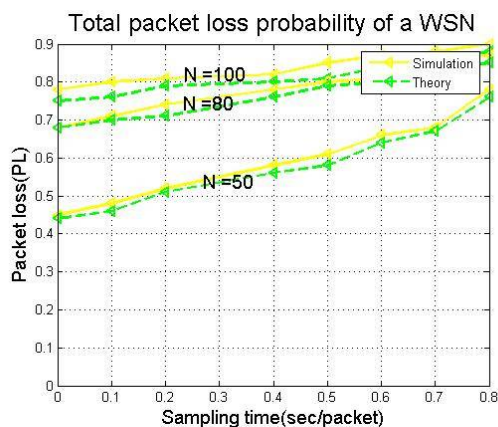


Fig. 7 Total Packet loss probability of a WSN

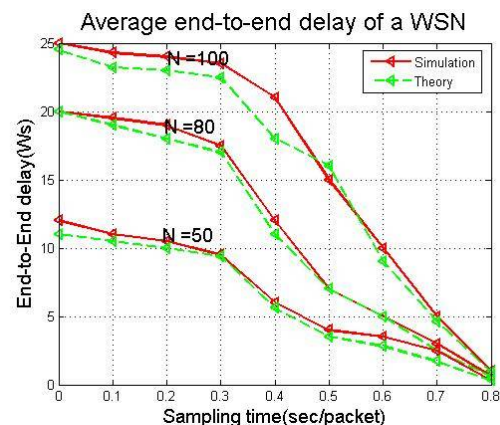


Fig. 8 Average end-to-end delay of a WSN

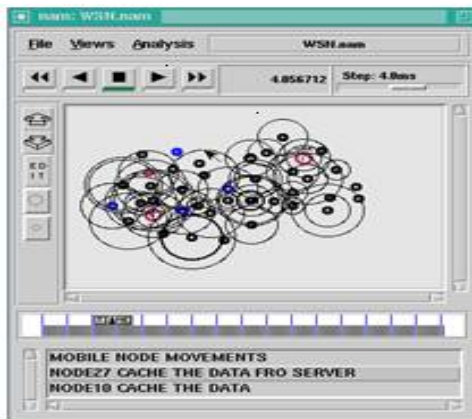


Fig. 9 finite buffer nodes without loss of packets

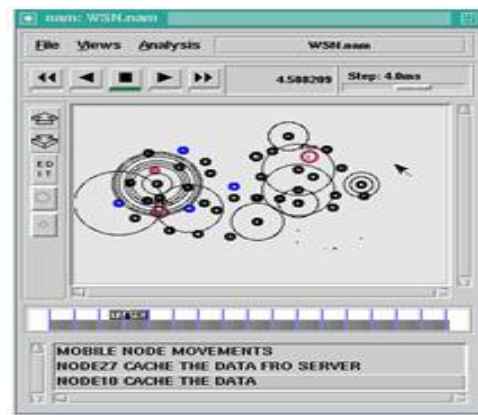


Fig. 10. Finite buffer nodes with loss of packets

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