ORDER REDUCTION USING POLE CLUSTERING AND MIXED MATHEMATICAL METHOD

A Pramod, P Bhargavi**, P Surya Surekha***, M Ravi****, B Praveen Kumar******
*K Gowri*******
Department of EEE
R Ganesh, Assistant Professor, Department of EEE,
Avanthi Institute of Engineering & Technology, Narsipatnam, bganesheee@gmail.com
*pramodanapareddy@gmail.com *, bhargavinrpm@gmail.com **, surya3539s@gmail.com ***,*
*meesalaravi19@gmail.com ****, borrapraveen804@gmail.com *****, gowrikalla210@gmail.com *******

Abstract: In this paper a method is proposed for finding stable reduced order models of single-input-single-output large scale systems using Mixed Mathematical Method and the Classic Dominance Pole Clustering technique. The denominator polynomial of the reduced order model with respect to original model is determined by forming the clusters of the numerator polynomial with respect to original model are obtained by using the mixed mathematical method. The mixed methods are simple and guarantee the stability of the reduced model if the original system is stable. The methodology of the proposed methods illustrated with the help of examples from literature

Keywords: Classic Dominance Pole Clustering, Order reduction, Factor Division, Transfer Function

I. INTRODUCTION

A great number of problems are brought about by the present day technology and societal and environmental process which are highly complex and large in dimensions and stochastic in nature. There has been no accepted definition for what constitute a large scale system. Many viewpoints have been presented on this issue. One view point has been that a system is considered large scale if it can be decoupled or partitioned in to number of interconnected subsystems or small scale systems for either computational or practical reasons. Another view points is that a system is large scale when its dimensions are so large that conventional techniques of modeling, analysis, control, design and computational fail to give reasonable solutions with reasonable computational efforts In other words a system is large when it requires more than one controller. Since the early 1950s,when classical control theory was being established, engineers have devised several procedures, both within the classical and modern control contexts, which analyze or design a given system. These procedures can be summarized as follows.

1. Modelling procedures which consist of differential equations, input-output transfer functions and state space formulations.
2. Behavioral procedures of systems such as controllability, observability and stability tests and application of such criteria as Routh-Hurwitz, Nyquist, Lyapunov's second method etc.,
3. Control procedures such as series compensation, pole placement, optimal control etc., The underlying assumption for all such control and system procedures has been centrality i.e., all the calculations based upon system information and the information itself are localized at a given center, very often a geographical position.

Necessity of Model Reduction: In practical situations most of the systems are of very high order so, their exact analysis and design become both tedious and costly. In order to perform simulation analysis or control design on those higher order models one will face many difficulties. It is desirable to represent those physical systems with reduced order model that will resemble the original model in time and frequency domain.

II. CONVENTIONAL TYPE OF REDUCTION METHODS

These are many methods available in literature for the reduction of higher order linear continuous time systems. The numerator and denominator can be solved by two different method. Some of the familiar methods available for the reduction of higher order continuous time system and studies for the thesis work are

1. Moment matching method,
2. Continued fraction method,
3. Stability equation method,
4. Routh – approximation method,
5. Interpolation method,
6. Pade approximation method.

Aim of this Work:

- To study some important and recently developed model reduction technique available in literature for reduction of high order SISO continuous time system.
- To consider and verify the some of the existing methods of model reduction techniques available in the literature for the high order SISO continuous time systems.
- A mixed method is considered for reduction of higher order SISO time system using pade approximation and standard characteristics of original higher order system which overcome the drawback of existing methods available in the literature.

III. MODEL REDUCTION METHODS FOR HIGHER ORDER CONTINOIUS TIME SYSTEMS

1. ROUTH – APPROXIMATION METHOD (RAM):

REDUCTION PROCEDURE:

Consider the transfer function of nth order original systems as:

$$G(s)=\frac{A_1 s^{n-1}+A_2 s^{n-2}+\cdots\ldots+A_n}{B_0 s^n+B_1 s^{n-1}+\cdots\ldots\ldots+B_n}$$

The above equation can be written in the following canonical form

$$G(s)=\beta_1 f_1(s)+\beta_2 f_1(s) f_2(s)+\cdots\ldots\ldots+\beta_n f_1(s)+\cdots\ldots f_n(s)$$

$$G(s)=\sum_{i=1}^{n}\beta\prod_{j=1}^{i}f_j(s)$$

Where β_i i=1, 2….nand, $f_k(s)$,k=2, 3….n are determined by the continued fraction

$$f_k(s) = \cfrac{1}{\alpha_k s + \cfrac{1}{\alpha_{k+1} s + \cfrac{1}{\alpha_{k+2} s + \cfrac{1}{\cdots\cdots \alpha_{n-1} s + \cfrac{1}{\alpha_n s}}}}}$$

$$\text{And } f_1(s) = \frac{1}{1+\alpha_1 s}$$

Equations are called alpha-beta expansions of G(s). $\alpha - \beta$ Parameters can be obtained by the following tables.

α-Table:

$b_0^0 = b_0$ $b_0^1 = b_1$	$b_2^0 = b_2$ $b_2^1 = b_3$	$b_4^0 = b_4$ $b_4^1 = b_5$	$b_6^0 = b_6$ $\ldots$	$\ldots$
$b_2^0 = b_2^0 - \alpha_1 b_2^1$ $b_3^0 = b_2^1 - \alpha_2 b_2^2$ $b_4^0 = b_2^2 - \alpha_3 b_2^3$ $b_0^5 = b_2^3 - \alpha_4 b_2^4$	$b_2^2 = b_4^0 - \alpha_1 b_4^1$ $b_2^3 = b_4^1 - \alpha_2 b_4^2$ $b_2^4 = b_4^2 - \alpha_3 b_4^3$ ………………	$b_4^2 = b_6^0 - b_6^1$ …………… ……………		

β-Table:

$a_0^1 = a_1$ $a_0^2 = a_2$	$a_2^1 = a_3$ $a_2^2 = a_4$	$a_4^1 = a_5$ $a_4^2 = a_6$
$a_0^3 = a_2^1 - \beta_1 b_2^1$	$a_2^3 = a_4^1 - \beta_1 b_4^1$	……………
$a_0^4 = a_2^2 - \beta_2 b_2^2$	$a_2^4 = a_4^2 - \beta_2 b_4^2$	……………
$a_0^5 = a_2^3 - \beta_3 b_2^3$	…………	
$a_0^6 = a_2^4 - \beta_4 b_2^4$	…………	

The alpha parameters are defined as

$\alpha_i = \frac{b_0^{i-1}}{b_0^i}$, i=1,2,3……..n

The beta parameters are defined as

$\beta_i = \frac{a_0^i}{b_0^i}$, i=1,2,3……..

The numerator and denominator of the k th order reduced model $(R_k(s))$ can be expressed in $\alpha - \beta$s are as follows.

Reduced Order Numerator:

$$p_1(s) = \beta_1, p_2(s) = \beta_2 + \alpha_2\beta_1 s$$

The general expression for the numerator is

$$p_k(s) = \alpha_k s p_{k-1}(s) + p_{k-2}(s) + \beta_k$$

Where k=1,2,……….and $p_{-1}(s) = p_0(s) = 0$

Reduced Order Denominator:

$$Q_k(s) = \alpha_k s Q_{k-1}(s) + Q_{k-2}(s)$$

Where k=1,2,………. And $Q_{-1} = Q_0(s) = 1$

The reduced model preserves high frequency characteristics and for control application it is preferable to use reciprocal transfer function defined by

$$G(s)=\frac{1}{s}G\left(\frac{1}{s}\right) = \frac{a_n s^{n-1} + \cdots \ldots a_2 s + a_1}{b_n s^n + \cdots \ldots + b_1 s + b_0}$$

Algorithm:

Determine the reciprocal transformation of the given original system $\hat{G}(s)$.

1. Construct α-β tables corresponding to $\hat{G}(s)$ from the table-2 and table-3 respectivel
2. Find out numerator and denominator for kth order reduced model by using equations.

$$p_k(s) = \alpha_k s p_{k-1}(s) + \beta_k, Q_k(s) = \alpha_k(s) Q_{k-1}(s) + Q_{k-2}(s)$$

Where k=1, 2,……………and $P_{-1}(s) = P_0(s) = 0, Q_{-1}(s) = Q_0(s) = 1$

3. Constitute reduced order model as

$$R_k(s) = \frac{P_k(s)}{Q_k(s)}$$

4. Any reciprocal transformation on $R_k(s)$ and to obtain $R_k(s)$

$$R_K(S) = \frac{P_K(S)}{Q_K(S)}$$

NUMERICAL EXAMPLE:

$$G(s) = \frac{40320 + 185760S + 222088S^2 + 122664S^3 + 36380S^4 + 5982S^5 + 514S^6 + 18S^7}{40320 + 109584S + 118124S^2 + 67284S^3 + 22449S^4 + 4536S^5 + 546S^6 + 36S^7 + S^8}$$

Replace S by $\frac{1}{S}$

$$G(s) = \frac{40320 + 185760(\frac{1}{S}) + 222088\left(\frac{1}{S}\right)^2 + 122664\left(\frac{1}{S}\right)^3 + 36380(\frac{1}{S})^4 + 5982(\frac{1}{S})^5 + 514(\frac{1}{S})^6 + 18(\frac{1}{S})^7}{40320 + 109584(\frac{1}{S}) + 118124\left(\frac{1}{S}\right)^2 + 67284\left(\frac{1}{S}\right)^3 + 22449\left(\frac{1}{S}\right)^4 + 4536\left(\frac{1}{S}\right)^5 + 546\left(\frac{1}{S}\right)^6 + 36\left(\frac{1}{S}\right)^7 + \left(\frac{1}{S}\right)^8}$$

$$G(s) = \frac{18 + 514S + 5982S^2 + 36380S^3 + 122664S^4 + 222088S^5 + 185760S^6 + 40320S^7}{1 + 36S + 546S^2 + 4536S^3 + 22449S^4 + 67284S^5 + 118124S^6 + 109584S^7 + 40320S^8}$$

From $\alpha - \beta$ method, we can find

$\alpha_1 = 0.367, \alpha_2 = 1.172 \ and \ \beta_1 = 0.3679, \beta_2 = 1.9882$

$A_2(s) = \alpha_1\alpha_2 S^2 + \alpha_2 S + 1 = 0.43S^2 + 1.172S + 1$

$$B_2(s) = \alpha_2\beta_1 S + \beta_2 = 0.4311S + 1.9882$$

$$R_2(s) = \frac{B_2(s)}{A_2(s)} = \frac{0.4311S + 1.9882}{0.43S^2 + 1.172S + 1} = \frac{0.4311\left(\frac{1}{S}\right) + 1.9882}{0.43\left(\frac{1}{S}\right)^2 + 1.172\left(\frac{1}{S}\right) + 1}$$

$$= \frac{0.4311 + 1.9882S}{0.43 + 1.172S + S^2}$$

2. MIXED MATHEMATICAL METHOD:

REDUCTION PROCEDURE:

Numerator is solved by using the pade approximation and standard characteristics of higher order equation.

Consider a function $f(s) = c_0 + c_1 s + c_2 s^2 + \cdots\cdots$

$$a_0 = b_0 c_0$$

$$a_1 = b_0 c_1 + b_1 c_0$$

$$a_2 = b_0 c_2 + b_1 c_1 + b_2 c_0$$

From this equations we get c_0, c_1, c_2,............ values.

Reducing the denominator by utilizng the characteristics of the system, like damping ratio (ξ), undamped natural frequency of oscillations (ωn) etc.

For an aperiodic or almost periodic system, ξ = 0.99, number of oscillations before the system settles = 1

Since, $\omega_n = 4/\xi{*}Ts$

Reduced denominator:

$$D_2(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$

By using the reduced denominator coefficients b_0, b_1, b_2. We are calculating a_0, a_1 values

The normalized reduced order

$$G(s)= \frac{a_0+a_1 s}{b_0+b_1 s+b_2 S^{\wedge}2}$$

NUMERICAL EXAMPLE:

$$G(s) = \frac{40320 + 185760S + 222088S^2 + 122664S^3 + 36380S^4 + 5982S^5 + 514S^6 + 18S^7}{40320 + 109584S + 118124S^2 + 67284S^3 + 22449S^4 + 4536S^5 + 546S^6 + 36S^7 + S^8}$$

Form the procedure we get,

C_0=1 and C_1=-1.8892

The reduced system $= \frac{a_0+a_1 s}{s^2+2\varepsilon\omega_n s+{\omega_n}^2}$

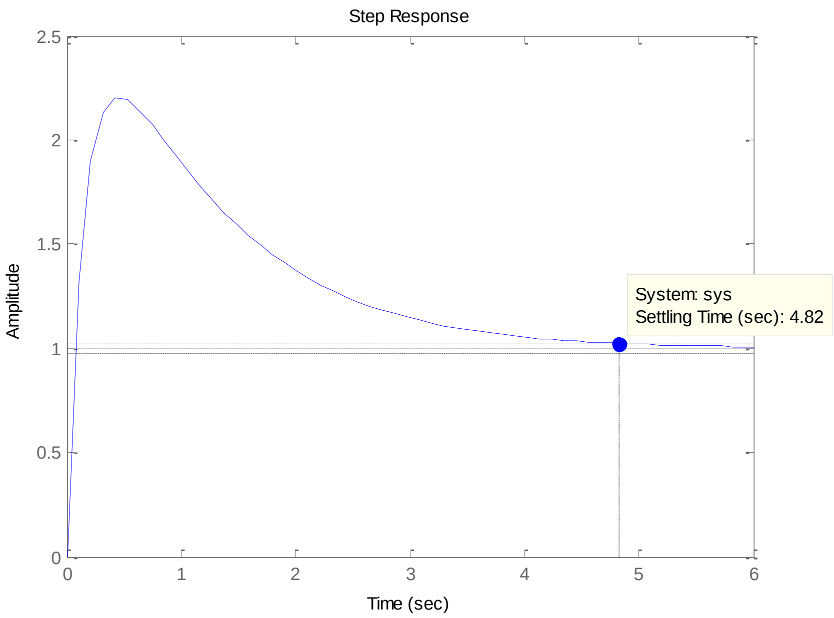

Figure 1: Step response of original higher order system

In these ξ =0.99

From the above figure settling time is

T_s=4.82, ω_n=08382

The reduced denominator is $D(s) = S^2 + 1.6596S + 0.7$

here b_0=0.7, b_1=1.6596

The reduced order transfer function is

$$R_2(s) = \frac{0.7 + 2.9871S}{0.7 + 1.6596S + S^2}$$

3 CLASSIC DOMINANCE POLE CLUSTERING METHOD:

Let the transfer function of the original high order linear dynamic SISO system of order ‘n’ be:

$$G_n(s) = \frac{N(S)}{D(S)} = \frac{b_0 + b_1 S + \cdots + b_{n-1} s^{n-1}}{a_0 + a_1 S + \cdots + a_{n-1} s^{n-1} + a_n s^n}$$

and Let the corresponding rth order reduced model is synthesized as:

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_0 + d_1 s + \cdots + d_{r-1} s^{r-1}}{e_0 + e_1 s + \cdots + e_{r-1} s^{r-1} + s^r}$$

Further, the method consists of the following steps.

Step 1: Determination of the reduced order denominator polynomial with an improved pole clustering technique:

Calculate the 'n' number of poles from the given higher order system denominator polynomial. The number of cluster centres to be calculated is equal to the order of the reduced system. The poles are distributed in to the cluster centre for the calculation such that none of the repeated poles present in the same cluster centre. Minimum number of poles distributed per each cluster centre is at least one. There is no limitation for the maximum number poles per cluster centre. Let '*k'* number of poles available in a cluster centre: $p_1, p_2, p_3 \ldots p_k$. The poles are arranged in a manner such that $|p_1| < |p_2| < \cdots < |p_k|$. The cluster center for the reduced order model can be obtained by using the following procedure.

1. Let 'k' number of poles available are $|p_1| < |p_2| < \cdots |p_k|$.
2. Set L=1; $C_L = \left[\left(\frac{-1}{|p_1|} + \sum_{i=2}^{K} \frac{-1}{|p_i - p_1|}\right) \div k\right]^{-1}$
3. Set L=L+1
4. Set Pole Cluster Centre

   $C_L = -\sqrt{p_1 * C_{L-1}}$
5. Check for L=K. if yes, then the final cluster center is $C_C = C_L$ and terminates the process. Otherwise proceed on to next step.
6. Check for L=K. if no, then go to step5. Otherwise go the next step.
7. Final cluster center is $C_C = C_L$ On calculating the cluster center values, we have following as in.

Case 1: All the denominator poles are real:

The corresponding reduced order denominator polynomial can be obtained as,

$$D_r(s) = (S - C_{C1})(S - C_{C2}) \ldots (S - C_{cr})$$

Where $c_{c1}, c_{c2} \ldots c_{cr}$ are the improved cluster values required to obtain the reduced order denominator polynomial of order r'

Case 2: All the poles are complex:

Let t = $(\frac{k}{2})$ pairs of complex conjugate poles in a L^{th} cluster be ,

$[(\sigma_1 \pm \omega_1), (\sigma_2 \pm \omega_2), (\sigma_3 \pm \omega_3) \ldots (\sigma_t \pm \omega_t)]$

Where,$|\sigma_1| < |\sigma_2| < \cdots |\sigma_L|$.apply the proposed algorithm individually for real and imaginary parts to obtain the respective improved cluster centers. The improved cluster is theform of $\mu_j = A_j \pm B_j$.Where,A_j *and* B_j is the improved pole cluster values obtained for real and imaginary parts respectively.

The corresponding reduced order denominator polynomial can be obtained as,

$D_r(s) = (s + |\mu_1|)(s + |\mu_2|) \ldots (s + |\mu_j|)$.Where j=r

Case 3:

If some poles are real and some poles are complex in nature ,applying an improved clustering algorithm separately for real and complex terms .Finally obtained improved cluster centers are combined together to get the reduced order denominator polynomial.

4 FACTOR DIVISION METHOD:

Determination of the numerator of kth order reduced model using factor division algorithm. After obtaining the reduced denominator, the numerator of the reduced model is determined as follows.

$$N(s)= \frac{N(s)}{D(s)} * D_k(s) = \frac{N(s)}{D(S)/D_k(s)}$$

Where $D_K(s)$ is reduced order denominator

There are two approaches for determining of numerator of reduced order model.

1. By performing the product of N8(s) and Dk(s) as the first row of factor division algorithm and D8(s) as the second row up to sk-1 terms are needed in both rows.
2. By expressing N8(s)Dk(s)/D8(s) as N8(s)/[D8(s)/Dk(s)] and using factor division algorithm twice ;the first time to find the term up to sk-1 in the expression of D8(s)/Dk(s) (i.e. put D8(s) in the first row and Dk(s) in the second row, using only terms up to s^k-1), and second time with N8(s) in the first row and expansion [D8(s)/Dk(s)] in the second row.

Therefore the numerator Nk(s) of the reduced order model will be the series expansion of

$$\frac{N_g(s)}{\frac{D_g(s)}{D_k(s)}} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k-1} d_i s^i}$$

About s=0 up to term of order s^k-1.

This is easily obtained by modifying the movement generating .which uses the familiar routh recurrence formula to generate the third , fifth , seventh etc rows as

$$\propto_0 = \frac{d_0}{f_0} < \begin{matrix} d_0 & d_1 \ldots\ldots\ldots d_{k-1} \\ f_0 & f_1 \ldots\ldots\ldots f_{k-1} \end{matrix}$$

$$\propto_1 = \frac{g_0}{f_0} < \begin{matrix} g_0 & g_1 \ldots\ldots\ldots g_{k-2} \\ f_0 & f_1 \ldots\ldots\ldots f_{k-2} \end{matrix}$$

$$\propto_2 = \frac{l_0}{f_0} < \begin{matrix} l_0 & l_1 \ldots\ldots\ldots l_{k-3} \\ f_0 & f_1 \ldots\ldots\ldots f_{k-3} \end{matrix}$$

…......................

…......................

$$\propto_{k-2} = \frac{p_0}{f_0} < \begin{matrix} p_0 & p_1 \\ f_0 & f_1 \end{matrix}$$

$$\propto_{k-2} = \frac{q_0}{f_0} < \begin{matrix} q_0 \\ f_0 \end{matrix}$$

Where

$$g_i = d_{i+1} - \propto_0 * f_{i+1} \quad i = 0,1,2, \ldots \ldots.$$

$$l_i = g_{i+1} - \propto_1 * f_{i+1} \quad i = 0,1,2, \ldots \ldots \ldots$$

…................................

…................................

$$l_o = p_1 - \propto_{k-2} * f_1$$

Therefore the numerator $N_m(s)$ is given by $N_k(s) = \sum_{i=0}^{k-1} \propto_i s^i$

NUMERICAL EXAMPLE

$$G(s) = \frac{40320 + 185760S + 222088S^2 + 122664S^3 + 36380S^4 + 5982S^5 + 514S^6 + 18S^7}{40320 + 109584S + 118124S^2 + 67284S^3 + 22449S^4 + 4536S^5 + 546S^6 + 36S^7 + S^8}$$

the poles are: -1, -2, -3, -4, -5, -6, -7, -8

Let the 2nd order reduced model is required to be realized, for this purpose only two real clusters are required.

Let the first and second cluster consists the poles (-1, -2, -3, -4) and (-5,-6,-7,-8) respectively. The modified cluster centers are computed as:

For (-1,-2,-3,-4)

Set L=1; C_1=-1.4119

Set L=L+1, the Modified Cluster center is C_2=-1.18824

Set L=L+1, the Modified Cluster center is C_3=-1.09006

Set L=L+1, the Modified Cluster center is **C_4=-1.04406**(final Cluster)

For -5,-6,-7,-8)

Set L=1; C_1=-1.9675

Set L=L+1, the Modified Cluster center is C_2=-3.1365

Set L=L+1, the Modified Cluster center is C_3=-3.9601

Set L=L+1, the Modified Cluster center is **C_4=-4.4497**(final Cluster)

The denominator polynomial $D_2(s)$ is obtained as;

$$D_2(s) = (s - P_{c1})(s - P_{c2}) = \big(s - (-1.04406)\big)\big(s - (-4.4497)\big)$$
$$= 4.645 + 5.4937S + S^2$$

Determination of the Numerator of Kth order reduced model using Factor Division Algorithm

$$\tilde{N}(s) = \frac{N(s)}{D(s)} * D_k(s) = \frac{N(s)}{D(s)\big/D_k(s)}$$

Where $D_k(s)$ is reduced order denominator

The Reduced Model is given as:

$$R_2(s) = \frac{4.645 + 14.268s}{4.645 + 5.4937s + s^2}$$

IV. RESULTS

Step Response Of Higher & Equivalent Reduced Order Systems for the following Example

Let us consider higher order equation as

$$G(s) = \frac{40320 + 185760S + 222088S^2 + 122664S^3 + 36380S^4 + 5982S^5 + 514S^6 + 18S^7}{40320 + 109584S + 118124S^2 + 67284S^3 + 22449S^4 + 4536S^5 + 546S^6 + 36S^7 + S^8}$$

Reduction Method	Reduced Order System
Classic Dominance Pole Clustering Method(CDPCM)	$\frac{14.268S + 4.645}{S^2 + 5.493S + 4.645}$
Routh Approximation Method $(\alpha - \beta)$	$\frac{1.9882S + 0.4311}{S^2 + 1.172S + 0.43}$
Factor Division Method	$\frac{14.269S + 4.645}{S^2 + 5.4937S + 4.645}$
Mixed Mathematical Method	$\frac{2.9820S + 0.7}{S^2 + 1.6596S + 0.7}$

Table 1: Equivalent Reduced Order Systems for Example

Time Domain Specifications Of Original And Reduced Order Systems for Example

Time Domain Specifications	Original System	Routh Approximation	Classic Dominance Pole	Mixed Method	Factor Division

		Method $(\alpha - \beta)$	Clustering (CDPCM)		Method
Peak amplitude	2.2	1.57	2.29	1.65	2.29
Overshoot	120	57	129	64.8	129
Peak time	0.424	2.35	0.508	1.66	0.508
Settling time	4.82	8.76	4.52	7.93	4.52
Rise time	0.0644	0.555	0.0724	0.364	0.0724
Final value	1	1	1	1	1

Table 2: Comparison of a Time Domain Specifications of Higher & Reduced Order Systems for Example

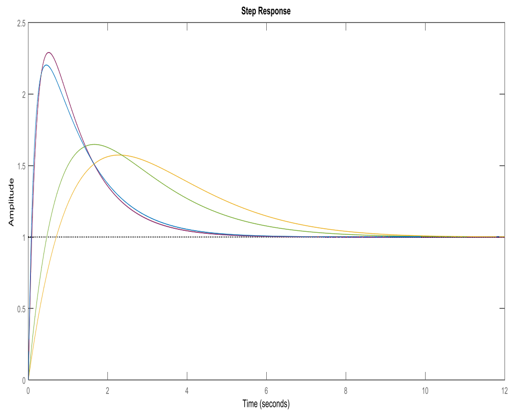

Figure 2: Comparison of Step Response of Higher Order System & Equivalent Reduced Order Systems for Example

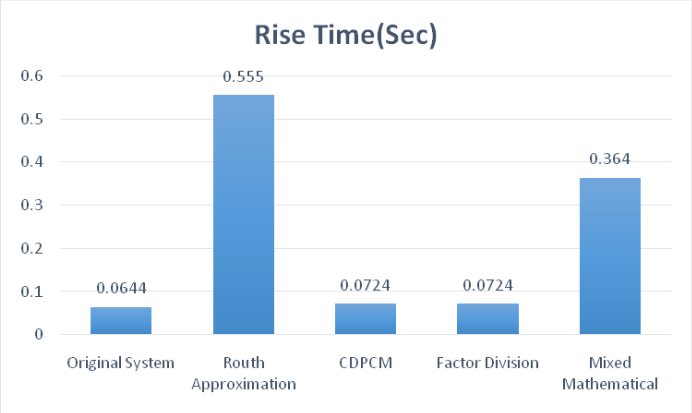

Figure 3: Variation in Rise Time with Different Methods

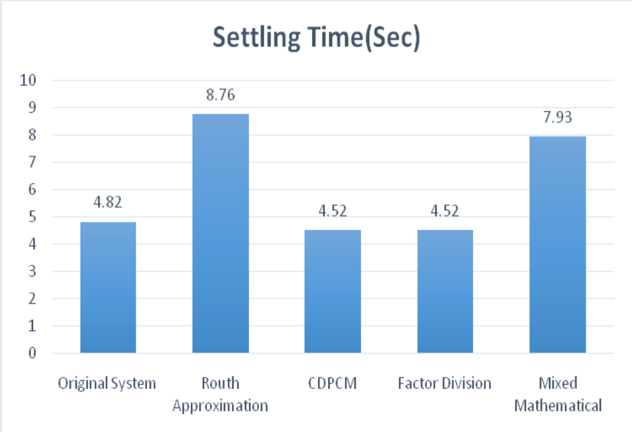

Figure 3: Variation in Settling Time with Different Methods

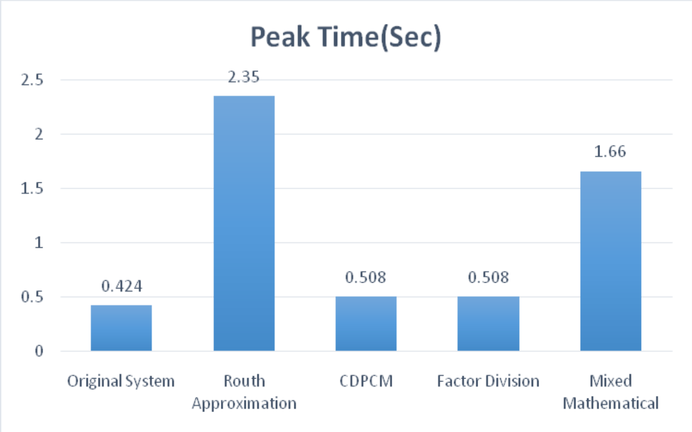

Figure 4: Variation in Peak Time with Different Methods

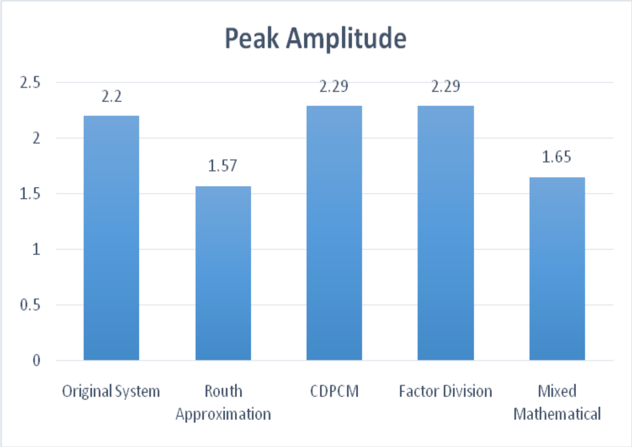

Figure 5: Variation in Peak Amplitude with Different Methods

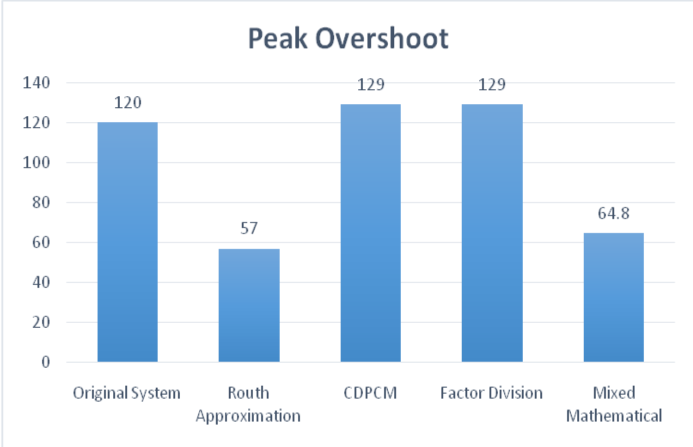

Figure 6: Variation in Peak Overshoot with Different Methods

From the above Tables by comparing time domain specifications of reduced order systems with higher order system, definitely reduced order system has more advantages in terms same final value, less peak overshoot, less settling time, less peak amplitude and faster rising time. Whereas in reduced order systems with different reduction techniques having mixed advantages. From the table, observed that

- Routh Approximation and Mixed method has less peak amplitude compared to remaining methods.
- Classic Dominance Pole Clustering method has faster rise time when compared to original and remaining methods.
- Classic dominance Pole Clustering method has less settling time compared to remaining methods but having higher peak overshoot when compared to other Method.

So from the above observations, concluded that Pole Clustering Method has mixed advantages like less settling time, peak time and rise time compared to all the methods and Original higher order system.

V. CONCLUSION

From the above results we are proposed an order reduction method for the linear single-input-single-output higher order systems. The determination of the polynomial of the reduced model is done by using the Classic dominance pole clustering method while the numerator coefficients are computed by factor division method. The merits of proposed method are stable, simplicity, efficient and computer oriented. The proposed algorithm has been explained with an example taken from the literature. The step responses of the original and reduced system of second order are shown in the Fig (2-6). A comparison of proposed method with the other well known order reduction methods in the literature is shown in the Table from which we can concluded that proposed method is comparable in quality.

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